

# Package: RMTstat (via r-universe)

September 16, 2024

**Version** 0.3.1

**Date** 2022-04-12

**Title** Distributions, Statistics and Tests Derived from Random Matrix Theory

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**Description** Functions for working with the Tracy-Widom laws and other distributions related to the eigenvalues of large Wishart matrices. The tables for computing the Tracy-Widom densities and distribution functions were computed by functions were computed by Momar Dieng's MATLAB package ``RMLab''. This package is part of a collaboration between Iain Johnstone, Zongming Ma, Patrick Perry, and Morteza Shahram.

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**Encoding** UTF-8

**URL** <https://github.com/evanbiederstedt/RMTstat>

**BugReports** <https://github.com/evanbiederstedt/RMTstat>

**Repository** <https://evanbiederstedt.r-universe.dev>

**RemoteUrl** <https://github.com/evanbiederstedt/rmtstat>

**RemoteRef** HEAD

**RemoteSha** 0917aa200aa6e4168a413012da146406d706d246

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**Description**

Density, distribution function, quantile function and random generation for the Marčenko-Pastur distribution, the limiting distribution of the empirical spectral measure for a large white Wishart matrix.

**Usage**

```
dmp( x, ndf=NA, pdim=NA, var=1, svr=ndf/pdim, log = FALSE )
pmp( q, ndf=NA, pdim=NA, var=1, svr=ndf/pdim, lower.tail = TRUE, log.p = FALSE )
qmp( p, ndf=NA, pdim=NA, var=1, svr=ndf/pdim, lower.tail = TRUE, log.p = FALSE )
rmp( n, ndf=NA, pdim=NA, var=1, svr=ndf/pdim )
```

**Arguments**

x, q	vector of quantiles.
p	vector of probabilities.
n	number of observations. If <code>length(n) &gt; 1</code> , the length is taken to be the number required.
ndf	the number of degrees of freedom for the Wishart matrix.
pdim	the number of dimensions (variables) for the Wishart matrix.
var	the population variance.
svr	samples to variables ratio; the number of degrees of freedom per dimension.
log, log.p	logical; if TRUE, probabilities p are given as $\log(p)$ .
lower.tail	logical; if TRUE (default), probabilities are $P[X \leq x]$ , otherwise, $P[X > x]$ .

**Details**

The concentration can either be given explicitly, or else computed from the given `ndf` and `pdim`. If `var` is not specified, it assumes the default of 1.

The Marčenko-Pastur law is the limit of the random probability measure which puts equal mass on all `pdim` eigenvalues of a normalized `pdim`-dimensional white Wishart matrix with `ndf` degrees of freedom and scale parameter `diag(var, var, ..., var)`. It is assumed that `ndf` goes to infinity, and `ndf/pdim` goes to nonzero constant called the "samples-to-variables ratio" (`svr`).

**Value**

`dmp` gives the density, `pmp` gives the distribution function, `qmp` gives the quantile function, and `rmp` generates random deviates.

**Author(s)**

Iain M. Johnstone, Zongming Ma, Patrick O. Perry and Morteza Shahram

**Source**

Other than the density, these functions are relatively slow and imprecise.

The distribution function is computed with `integrate`. The quantiles are computed via bisection using `uniroot`. Random variates are generated using the inverse CDF.

**References**

Marčenko, V.A. and Pastur, L.A. (1967). Distribution of eigenvalues for some sets of random matrices. *Sbornik: Mathematics* **1**, 457–483.

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TracyWidom

*The Tracy-Widom Distributions*

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**Description**

Density, distribution function, quantile function, and random generation for the Tracy-Widom distribution with order parameter beta.

**Usage**

```
dtw(x, beta=1, log = FALSE)
ptw(q, beta=1, lower.tail = TRUE, log.p = FALSE)
qtw(p, beta=1, lower.tail = TRUE, log.p = FALSE)
rtw(n, beta=1)
```

**Arguments**

x, q	vector of quantiles.
p	vector of probabilities.
n	number of observations. If <code>length(n) &gt; 1</code> , the length is taken to be the number required.
beta	the order parameter (1, 2, or 4).
log, log.p	logical; if TRUE, probabilities p are given as <code>log(p)</code> .
lower.tail	logical; if TRUE (default), probabilities are $P[X \leq x]$ , otherwise, $P[X > x]$ .

**Details**

If beta is not specified, it assumes the default value of 1.

The Tracy-Widom law is the edge-scaled limiting distribution of the largest eigenvalue of a random matrix from the  $\beta$ -ensemble. Supported values for beta are 1 (Gaussian Orthogonal Ensemble), 2 (Gaussian Unitary Ensemble), and 4 (Gaussian Symplectic Ensemble).

**Value**

dtw gives the density, ptw gives the distribution function, qtw gives the quantile function, and rtw generates random deviates.

**Author(s)**

Iain M. Johnstone, Zongming Ma, Patrick O. Perry and Morteza Shahram

**Source**

The distribution and density functions are computed using a lookup table. They have been pre-computed at 769 values uniformly spaced between  $-10$  and  $6$  using MATLAB's `bvp4c` solver to a minimum accuracy of about  $3.4e-08$ . For all other points, the values are gotten from a cubic Hermite polynomial interpolation. The MATLAB software for computing the grid of values is part of RMLab, a package written by Momar Dieng.

The quantiles are computed via bisection using [uniroot](#).

Random variates are generated using the inverse CDF.

**References**

Dieng, M. (2006). Distribution functions for edge eigenvalues in orthogonal and symplectic ensembles: Painlevé representations. *arXiv:math/0506586v2 [math.PR]*.

Tracy, C.A. and Widom, H. (1994). Level-spacing distributions and the Airy kernel. *Communications in Mathematical Physics* **159**, 151–174.

Tracy, C.A. and Widom, H. (1996). On orthogonal and symplectic matrix ensembles. *Communications in Mathematical Physics* **177**, 727–754.

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WishartMax

*The White Wishart Maximum Eigenvalue Distributions*

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**Description**

Density, distribution function, quantile function, and random generation for the maximum eigenvalue from a white Wishart matrix (sample covariance matrix) with `ndf` degrees of freedom, `pdim` dimensions, population variance `var`, and order parameter `beta`.

**Usage**

```
dWishartMax(x, ndf, pdim, var=1, beta=1, log = FALSE)
pWishartMax(q, ndf, pdim, var=1, beta=1, lower.tail = TRUE, log.p = FALSE)
qWishartMax(p, ndf, pdim, var=1, beta=1, lower.tail = TRUE, log.p = FALSE)
rWishartMax(n, ndf, pdim, var=1, beta=1)
```

**Arguments**

x, q	vector of quantiles.
p	vector of probabilities.
n	number of observations. If <code>length(n) &gt; 1</code> , the length is taken to be the number required.
ndf	the number of degrees of freedom for the Wishart matrix
pdim	the number of dimensions (variables) for the Wishart matrix
var	the population variance.
beta	the order parameter (1 or 2).
log, log.p	logical; if TRUE, probabilities p are given as log(p).
lower.tail	logical; if TRUE (default), probabilities are $P[X \leq x]$ , otherwise, $P[X > x]$ .

**Details**

If beta is not specified, it assumes the default value of 1. Likewise, var assumes a default of 1.

A white Wishart matrix is equal in distribution to  $(1/n)X'X$ , where  $X$  is an  $n \times p$  matrix with elements i.i.d. Normal with mean zero and variance var. These functions give the limiting distribution of the largest eigenvalue from the such a matrix when ndf and pdim both tend to infinity.

Supported values for beta are 1 for real data and and 2 for complex data.

**Value**

dWishartMax gives the density, pWishartMax gives the distribution function, qWishartMax gives the quantile function, and rWishartMax generates random deviates.

**Author(s)**

Iain M. Johnstone, Zongming Ma, Patrick O. Perry and Morteza Shahram

**Source**

The functions are calculated by applying the appropriate centering and scaling (determined by [WishartMaxPar](#)), and then calling the corresponding functions for the [TracyWidom](#) distribution.

**References**

- Johansson, K. (2000). Shape fluctuations and random matrices. *Communications in Mathematical Physics*. **209** 437–476.
- Johnstone, I.M. (2001). On the ditribution of the largest eigenvalue in principal component analysis. *Annals of Statistics*. **29** 295–327.

**See Also**

[WishartMaxPar](#), [WishartSpike](#), [TracyWidom](#)

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WishartMaxPar

*White Wishart Maximum Eigenvalue Centering and Scaling*


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### Description

Centering and scaling for the maximum eigenvalue from a white Wishart matrix (sample covariance matrix) with `ndf` degrees of freedom, `pdim` dimensions, population variance `var`, and order parameter `beta`.

### Usage

```
WishartMaxPar(ndf, pdim, var=1, beta=1)
```

### Arguments

<code>ndf</code>	the number of degrees of freedom for the Wishart matrix.
<code>pdim</code>	the number of dimensions (variables) for the Wishart matrix.
<code>var</code>	the population variance.
<code>beta</code>	the order parameter (1 or 2).

### Details

If `beta` is not specified, it assumes the default value of 1. Likewise, `var` assumes a default of 1.

The returned values give appropriate centering and scaling for the largest eigenvalue from a white Wishart matrix so that the centered and scaled quantity converges in distribution to a Tracy-Widom random variable. We use the second-order accurate versions of the centering and scaling given in the references below.

### Value

<code>centering</code>	gives the centering.
<code>scaling</code>	gives the scaling.

### Author(s)

Iain M. Johnstone, Zongming Ma, Patrick O. Perry and Morteza Shahram

### References

El Karoui, N. (2006). A rate of convergence result for the largest eigenvalue of complex white Wishart matrices. *Annals of Probability* **34**, 2077–2117.

Ma, Z. (2008). Accuracy of the Tracy-Widom limit for the largest eigenvalue in white Wishart matrices. *arXiv:0810.1329v1 [math.ST]*.

### See Also

[WishartMax](#), [TracyWidom](#)

**Description**

Density, distribution function, quantile function, and random generation for the maximum eigenvalue from a spiked Wishart matrix (sample covariance matrix) with `ndf` degrees of freedom, `pdim` dimensions, and population covariance matrix `diag(spike+var, var, var, ..., var)`.

**Usage**

```
dWishartSpike(x, spike, ndf=NA, pdim=NA, var=1, beta=1, log = FALSE)
pWishartSpike(q, spike, ndf=NA, pdim=NA, var=1, beta=1, lower.tail = TRUE, log.p = FALSE)
qWishartSpike(p, spike, ndf=NA, pdim=NA, var=1, beta=1, lower.tail = TRUE, log.p = FALSE)
rWishartSpike(n, spike, ndf=NA, pdim=NA, var=1, beta=1)
```

**Arguments**

<code>x, q</code>	vector of quantiles.
<code>p</code>	vector of probabilities.
<code>n</code>	number of observations. If <code>length(n) &gt; 1</code> , the length is taken to be the number required.
<code>spike</code>	the value of the spike.
<code>ndf</code>	the number of degrees of freedom for the Wishart matrix.
<code>pdim</code>	the number of dimensions (variables) for the Wishart matrix.
<code>var</code>	the population (noise) variance.
<code>beta</code>	the order parameter (1 or 2).
<code>log, log.p</code>	logical; if TRUE, probabilities <code>p</code> are given as <code>log(p)</code> .
<code>lower.tail</code>	logical; if TRUE (default), probabilities are $P[X \leq x]$ , otherwise, $P[X > x]$ .

**Details**

The spiked Wishart is a random sample covariance matrix from multivariate normal data with `ndf` observations in `pdim` dimensions. The spiked Wishart has one population covariance eigenvalue equal to `spike+var` and the rest equal to `var`. These functions are related to the limiting distribution of the largest eigenvalue from such a matrix when `ndf` and `pdim` both tending to infinity, with their ratio tending to a nonzero constant.

For the spiked distribution to exist, `spike` must be greater than  $\sqrt{\text{pdim}/\text{ndf}} * \text{var}$ .

Supported values for `beta` are 1 for real data and 2 for complex data.

**Value**

`dWishartSpike` gives the density, `pWishartSpike` gives the distribution function, `qWishartSpike` gives the quantile function, and `rWishartSpike` generates random deviates.

**Author(s)**

Iain M. Johnstone, Zongming Ma, Patrick O. Perry and Morteza Shahram

**References**

- Baik, J., Ben Arous, G., and Pécché, S. (2005). Phase transition of the largest eigenvalue for non-null complex sample covariance matrices. *Annals of Probability* **33**, 1643–1697.
- Baik, J. and Silverstein, J. W. (2006). Eigenvalues of large sample covariance matrices of spiked population models. *Journal of Multivariate Analysis* **97**, 1382-1408.
- Paul, D. (2007). Asymptotics of sample eigenstructure for a large dimensional spiked covariance model. *Statistica Sinica*. **17**, 1617–1642.

**See Also**

[WishartSpikePar](#), [WishartMax](#)

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WishartSpikePar

*Spiked Wishart Eigenvalue Centering and Scaling*

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**Description**

Centering and scaling for the sample eigenvalue from a spiked Wishart matrix (sample covariance matrix) with `ndf` degrees of freedom, `pdim` dimensions, and population covariance matrix `diag(spike+var, var, var, . . . , var)`.

**Usage**

```
WishartSpikePar( spike, ndf=NA, pdim=NA, var=1, beta=1 )
```

**Arguments**

<code>spike</code>	the value of the spike.
<code>ndf</code>	the number of degrees of freedom for the Wishart matrix.
<code>pdim</code>	the number of dimensions (variables) for the Wishart matrix.
<code>var</code>	the population (noise) variance.
<code>beta</code>	the order parameter (1 or 2).

**Details**

The returned values give appropriate centering and scaling for the largest eigenvalue from a spiked Wishart matrix so that the centered and scaled quantity converges in distribution to a normal random variable with mean 0 and variance 1.

For the spiked distribution to exist, `spike` must be greater than  $\sqrt{\text{pdim}/\text{ndf}} * \text{var}$ .

Supported values for `beta` are 1 for real data and 2 for complex data.



**Value**

centering        gives the centering.  
scaleing        gives the scaling.

**Author(s)**

Iain M. Johnstone, Zongming Ma, Patrick O. Perry and Morteza Shahram

**References**

Baik, J., Ben Arous, G., and Pécché, S. (2005). Phase transition of the largest eigenvalue for non-null complex sample covariance matrices. *Annals of Probability* **33**, 1643–1697.

Baik, J. and Silverstein, J. W. (2006). Eigenvalues of large sample covariance matrices of spiked population models. *Journal of Multivariate Analysis* **97**, 1382-1408.

Paul, D. (2007). Asymptotics of sample eigenstructure for a large dimensional spiked covariance model. *Statistica Sinica* **17**, 1617–1642.

**See Also**

[WishartSpike](#)

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